Name _____ ID3 _ _ _

ECS 332: Principles of Communications **2018/1**

HW 3 — Due: September 26, 4 PM

Lecturer: Prapun Suksompong, Ph.D.

Instructions

(a) This assignment has 6 pages.

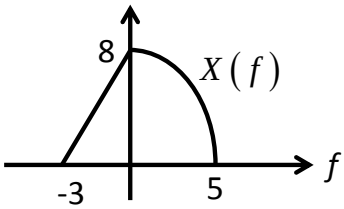
(b) (1 pt) Unless specified otherwise, work and write your answers **directly on these provided sheets** (not on other blank sheet(s) of paper). Hard-copies are distributed in class.

(c) (1 pt) Write your first name and the last three digits of your student ID on the upper-right corner of this page.

(d) (8 pt) Try to solve all non-optional problems.

(e) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1 (M2011)**.** The Fourier transform $X(f)$ for a signal $x(t)$ is shown in Figure 3.1.



$e^{j2\pi f_0 t} \longrightarrow \delta(f-f_0)$

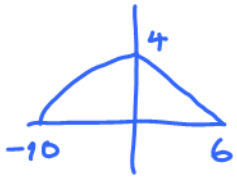
$g(t) = x(at)$

Figure 3.1: Plot of $X(f)$ for Problem 1.

Let $g(t) = x(-2t)$ and $y(t) = x(4-2t)$. Carefully sketch $|G(f)|$ and $|Y(f)|$.

$G(f) = \frac{1}{|a|} X\left(\frac{f}{a}\right)$

$a = -2 \rightarrow = \frac{1}{2} X\left(-\frac{f}{2}\right)$



$|G(f)| = \frac{1}{2}\left|X\left(-\frac{f}{2}\right)\right|$

expand horizontally by a factor of 2

flip horizontally

scale vertically by a factor of $\frac{1}{2}$

$y(t) = g(t-2) = x(-2(t-2))$

$= x(-2t+4)$

$Y(f) = e^{j2\pi(-2)f} G(f)$

$|Y(f)| = |G(f)|$

$z(t) = g(t-2) + g(t-3)$

$Z(f) = e^{j2\pi(-2)f} G(f) + e^{j(2\pi)(-3)f} G(f)$

Problem 2. [1]

(a) Consider the cosine pulse

$$p(t) = \begin{cases} \cos(10\pi t), & -1 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$$

(i) Sketch $p(t)$ for $-3 \le t \le 3$.

(ii) Find $P(f)$ analytically.

(iii) Sketch $P(f)$ from -10 Hz to 10 Hz.

[1] Inspired by [Carlson and Crilly, 2009, Q2.2-1 and Q2.2-2].

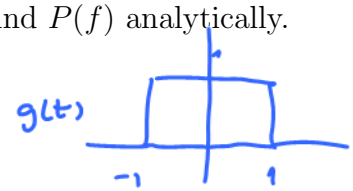
(b) Consider the cosine pulse

$$p(t) = \begin{cases} \cos(10\pi t), & 2 \le t \le 4 \\ 0, & \text{otherwise} \end{cases}$$

$= \cos(2\pi(5)t)$

$g(t-3) = x(t)$

(i) Find $P(f)$ analytically.



$\longrightarrow G(f) = 2\,\text{sinc}(2\pi f)$

$X(f) = e^{j2\pi(-3)f} G(f)$

$P(f) = \frac{1}{2}X(f-5) + \frac{1}{2}X(f+5)$

$= \cdots$

(ii) Use `MATLAB`. Mimic the code in `specrect.m` to plot the spectrum of $p(t)$. Follow the settings below:

- Consider the time t from 0 to 10 [s] when you set up the time vector.
- Use the sampling frequency of 500 samples per sec. So, the sampling interval (the time between adjacent samples) is $T_s = 1/500$.
- With the above sampling frequency, `plotspect` will plot the magnitude spectrum from -250 to 250 Hz. Use the function `xlim` (or the magnifier glass GUI) to limit your frequency view to be only from -10 to +10 Hz.

(iii) Also in `MATLAB`, add the plot of your analytical answer from part (i) into the same figure as part (ii).

- Print this figure and attach it at the end of your HW.
- On this attached page, compare the two plots. (Write some description/observation. Are they the same? How can you tell?)

Caution: The built-in `sinc` function in `MATLAB` is defined using the normalized version. So, you will need to remove a factor of π from the argument of each sinc function found in part (i) when you type it into `MATLAB`.

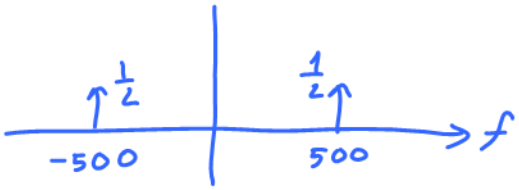
Problem 3. You are given the baseband signals (i) $m(t) = \cos 1000\pi t$; (ii) $m(t) = 2\cos 1000\pi t + \cos 2000\pi t$; (iii) $m(t) = (\cos 1000\pi t) \times (\cos 3000\pi t)$. For each one, do the following.

(a) Sketch the spectrum of $m(t)$.

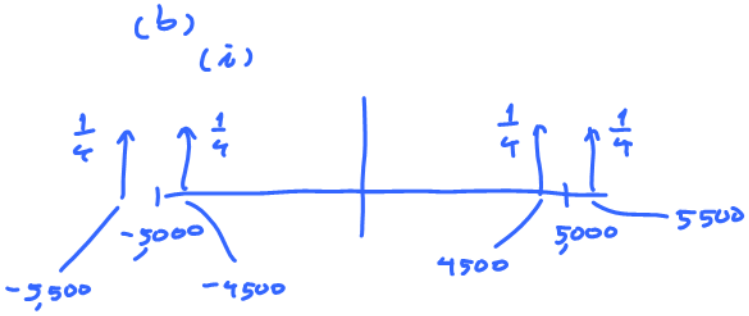
(b) Sketch the spectrum of the DSB-SC signal $m(t)\cos(10,000\pi t)$.

[Lathi and Ding, 2009, Q4.2-1]

(a) (i)



(b) (i)



Problem 4. Given a system with input-output relationship of

$$y(t) = 2x(t) + 10,$$

is this system linear? [Carlson and Crilly, 2009, Q2.3-10]

Problem 5. Signal $x(t) = 10\cos(2\pi \times 7 \times 10^6 \times t)$ is transmitted to some destination. The received signal is $y(t) = 10\cos(2\pi \times 7 \times 10^6 \times t - \pi/6)$.

(a) What is the minimum distance between the source and destination?

$y(t) = x(t - \tau)$ $\quad \tau = \frac{d}{c} = \frac{d}{3 \times 10^8}$

(b) What are the other possible distances?

[Carlson and Crilly, 2009, Q2.3-14]

Problem 6 (M2011)**.** Use properties of Fourier transform to evaluate the following integrals. (Do not integrate directly. Recall that $\text{sinc}(x) = \frac{\sin(x)}{x}$.) Clearly state the property or properties that you use.

(a) $\int_{-\infty}^{\infty} \text{sinc}\left(\sqrt{5}x\right)dx$

$g(t)|_{t=a}$ → 

(b) $\int_{-\infty}^{\infty} \text{sinc}\left(\sqrt{5}x\right)\text{sinc}\left(\sqrt{7}x\right)dx$

(c) (Optional) $\int_{-\infty}^{\infty} e^{-2\pi f\times 2j}2\text{sinc}\left(2\pi f\right)\left(e^{-2\pi f\times 5j}2\text{sinc}\left(2\pi f\right)\right)^{*}df$

(d) (Optional) $\int_{-\infty}^{\infty} \text{sinc}\left(\pi\left(x-5\right)\right)\text{sinc}\left(\pi\left(x-\frac{7}{2}\right)\right)dx$